Report to CASI on Improvements to the Mathematics of the Sporting Code

CASI Mathematics Working Group
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Executive Summary

As a result of the 2012 changes to Chapter 7 of the General Section, a mathematical paradox appeared. The conclusion of this study, commissioned by CASI, is that the level of mathematical sophistication of the Chapter is inappropriate for a modern international federation, and that the Chapter should be reorganised to support established practice and to meet the computational needs of all the Air Sports.

Contents

History

Chapter 7 of the General Section (GS7), "Measurement Requirements," has evolved over the years.

Before 1990 GS7 specified that the earth model to be used for distance calculations was the "FAI Sphere," a sphere of radius 6378.245 kilometres. Great Circle calculations were to be used in the determination of credited distances.

In 1990 the radius was changed to 6371.0 kilometres.

In 1994, an ellipsoid (6378.1379 by 6356.7523 kilometres) was introduced. This is the same ellipsoid as the one specified in the WGS84 Geodetic Datum, and it has since been known as the "WGS84 Ellipsoid." At the time of the introduction of the ellipsoid, GS7 did not disallow the use of any earth model for calculations.

In 2012, the General Section was amended, and GS 7.3.1.1 was changed to read, "The earth model for FAI purposes is the WGS84 ellipsoid." Corresponding changes were also made in the General Section Glossary.

Problem Description

The 2012 amendment caused an immediate computational problem for a majority of Air Sports.

Our analysis of that problem highlighted other mathematical and logical shortcomings of Chapter 7 and the Glossary. We have concluded that the immediate problem must be treated within the context of Chapter 7 and the Glossary. Consequently, a proper description of the problem requires some explanation. We have divided the problem description into three parts:

1. The immediate effect of the 2012 amendment ("AL16") on Air Sports
2. The state of the General Section Glossary
3. The organisation of General Section Chapter 7

Problem Description Part 1 – The immediate effect of AL16 on Air Sports

The 2012 amendment ("AL16") has created a new situation for several Air Sports, and, from a mathematical point of view, a new problem for all Air Sports.

- The new situation is that the seven Air Sport Commissions that currently depend on earth models other than the WGS84 ellipsoid are now out of compliance with the General Section (specifically, GS 7.3.1.1).

- The new problem is that compliance with the new GS 7.3.1.1 is computationally infeasible.

These two points are elaborated below.

Seven Air Sports are out of compliance

The Air Sport Commissions affected are CIA, CIMA, CIVL, IGC, GAC, CIG, and ICARE. Appendix A of this Report lists some of the computer programs currently used by the Commissions, and the earth models implemented in those programs.

Air Sports use different earth models for different purposes. Most Air Sports require the WGS84 ellipsoid for the calculation of international records. The exceptions are Ballooning and Space Records:

- In Ballooning, while total distances can be very large, the credited distance for records is calculated by summing short segments whose definition depends on a spherical coordinate system.

- For the calculation of space records, the reference surface is the top of the atmosphere (not the bottom). This is an ellipsoid, but it is not the WGS84 ellipsoid.

Furthermore, these variances from the ellipsoidal earth model currently exist:

- In Hot Air Ballooning and Microlight competitions, performances are sometimes measured directly from flat maps. The maps are approved by the ASC concerned and may or may not be projections of the WGS84 Geodetic Datum. In Gas Balloon competitions, distance calculations are made using the recently-disapproved FAI Sphere model.

- In Microlight Aviation, Rotorcraft, and General Aviation, the competition tasks may depend on a strict Cartesian coordinate system, with orthogonal axes. A flat plane earth model is used to set these tasks. The same model is used to evaluate flight performances with respect to these tasks, and to calculate scores.

- In General Aviation, Gliding, and Rotorcraft, some aspects of flight evaluation are done by inspection of flight paths as depicted on a computer screen. This is "pixel space," which is a flat plane earth model.

- Pixel space is also used by any Air Sport that depicts flight paths and maps on a web page, such as in online tracking.

<u>Compliance is computationally infeasible</u>

Strict adherence to an ellipsoidal earth model is, for all practical purposes, impossible. This is due to the mathematical fact that <u>no analytic geometry exists for the ellipsoid</u>. This means that, for geometric calculations on an ellipsoidal surface, approximations ("empirical methods") <u>must</u> be used. Useful empirical methods for the ellipsoid exist, but the set of methods available is incomplete. Due to the lack of availability of a complete set of empirical methods, the following situations exist:

- In Gliding, Hang Gliding, and Microlight Aviation competitions, calculation of distance and bearing to Start and Finish Lines or hidden gates may be required. This is practical only on a spherical or a flat plane earth model.

- In Ballooning and General Aviation, it is sometimes necessary to calculate the length of the "long path" (more than halfway around the world), a calculation that is not available on an ellipsoidal earth model.

- In all Air Sports, to avoid post-flight disputes, pilots and flight evaluators agree that onboard instruments should use the same algorithms that are used in pre-flight task definitions and in post-flight evaluations. In light of this, Air Sport Commissions have elected to use earth models that are mathematically complete, i.e. models that support analytical methods. Doing so eliminates:

a) the need to specify methods of approximation to the manufacturers of instruments and flight analysis programs; and

b) the need to verify that pilots, task setters, and evaluators are all using the same algorithms.

- In all Air Sports, the calculation of distance and bearing to airspace boundaries may be required. In the cases in which scoring depends on high precision (Gliding, Hang Gliding, Microlight Aviation), the calculation is practical only on a sphere or flat plane.

As stated above, there are no analytical geometric methods available for ellipsoidal earth models.

In the absence of analytic geometry, empirical methods must be used. All empirical methods give approximate answers. The one empirical method that we have at our disposal calculates distances between pairs of points - and is accurate for all practical aviation purposes. That's the good news.

The bad news is that we have no standard methods for calculations other than the calculation of the shortest distance between points. The classic example of this is that there is no practical way to solve the "cross-track error" problem on the ellipsoid (see Appendix B). Another example is that there is no way to calculate the "long path" on the ellipsoid.

In summary, the ellipsoidal earth model serves admirably when the objective is to calculate the shortest distance between two points, but it fails when other calculations are required. On the other hand, the spherical and flat plane earth models are mathematically complete, but fail to give accurate results over large distances.

Problem Description Part 2 – The state of the General Section Glossary

As would be expected, the changes to Chapter 7 caused by AL16 are supported by changes in the General Section Glossary. The following changes were marked as part of the amendment:

- The definition of *Geodesic* was reworded, and reference to the sphere was deleted.

- In the definition of *WGS84 Ellipsoid*, reference to the WGS84 Geodetic Datum was deleted.

And a deletion occurred:

- The definition of *Geodetic Datum* was removed.

The changes to the Glossary are consistent with the change to Chapter 7 caused by AL16.

(On a side note, the deletion of the definition of *Geodetic Datum* is completely baffling. FAI was one of the first aviation organisations in the world to recognise the need to standardise on a geodetic datum, and FAI led the way with its early adoption of WGS84).

Our review of the Glossary led us to the conclusion that the definitions therein are mathematically dubious, and at times excessively wordy and irrelevant (the latter two characterisations are admittedly non-mathematical opinions). Evidence in support of our opinion has existed since before AL16:

- A "three-dimensional ellipse" is mathematical nonsense (in the definition of *Ellipsoid*).

- The definition of *Geodesic* contains 97 words, but no definition of the term "geodesic."

- The definition of *FAI Sphere* describes it as an inferior concept, but contains no actual definition.

- The definition of *Sphere* contains no information at all.

- The Glossary entry *WGS84 Ellipsoid* does contain the definition of that ellipsoid. However, the story told in these 194 words makes very little sense without any definition of the WGS84 Geodetic Datum.

- The term *Earth Model* is used, but is never defined.

From a mathematical point of view, the General Section Glossary is far from rigorous. It is vague when it should be precise, and goes into unnecessary detail when a generalisation will do.

<u>Problem Description Part 3 – The organisation of General Section Chapter 7</u>

In addition to mathematical problems, General Section Chapter 7 has structural problems.

The Chapter has lost track of the difference between a measurement and a calculation. This occurred gradually over time, as the Chapter strayed from its original purpose, that of specifying measurement standards, to its apparent dual purpose of specifying measurement standards <u>and</u> calculation standards.

Mathematically, this is a problem because strict adherence to GS 7.3.1.1 is computationally infeasible.

Logically, this is a problem because Chapter 7 contradicts itself:

This paragraph

> 7.3.1 The methods and equipment for measuring and recording of Distance, Time, Speed, Altitude, Mass or other specific parameters, as well as equipment technical standards, **shall be determined by each FAI Air Sport Commission** and specified in the appropriate section of the Sporting Code *(emphasis added)*

is directly contradicted by the next paragraph:

> 7.3.1.1 The earth model for FAI purposes is the WGS84 ellipsoid

which appears to remove the authority of each Commission to determine how calculations are performed within its own area of responsibility.

Additionally, Chapter 7 has become disorganised. Over the years it has evolved from specifying "Measurement Requirements" to addressing photography, Observation Zones, record claim criteria, and geometric calculations. Some of these topics are relevant in our current era. Some are not.

The Chapter would benefit from a new organisation and a new title.

Proposed Solution

Overview

The immediate problem caused by AL16 could be solved by deleting that amendment and reverting to the pre-2012 wording of GS 7.3.1.1.

However, this simple approach would leave in place the unhelpful Glossary and the patchwork organisation of Chapter 7.

It is the opinion of this Working Group that all three problems could be solved by rewriting Chapter 7, with a view toward preserving its original purpose (measurement requirements), modernizing its new purpose (calculation requirements), and clarifying the distinction between the two.

To that end, we recommend the adoption of the wording that follows (in the next six pages of this Report) for Chapter 7 and the Glossary.

Notes about the proposed changes:

1. References to photography and Observation Zones have been deleted.

2. To preserve the context of our suggested new definitions, the entire Glossary is reproduced below.

3. New definitions and replacement definitions are in **bold.**

4. Because the Glossary pertains to the entire General Section, we have refrained from editing any Glossary entries that are not directly applicable to Chapter 7. However, in the version that follows, we have highlighted (without alteration) other portions of the Glossary that we consider to be outdated or problematical.

Proposed New Chapter 7

Measurements, Calculations, and Margins

7.1 Measurements

7.1.1 Units. The system of units to be used by FAI shall be the metric system (SI units), with the exception of angular units. Bearings shall be measured in degrees clockwise from True North. Coordinates shall be in units of degrees, with a preferred format of "degrees and decimal minutes."

7.1.2 General. The methods and standards of precision for measuring and recording of Position, Distance, Time, Altitude, Mass and other primary values, as well as equipment technical standards, shall be determined by each FAI Air Sport Commission and specified in the appropriate section of the Sporting Code. In the case of record flights, the conformity of the specific measuring and recording instruments and equipment used shall be checked by the Official Observer to be of the same type as approved by the respective FAI Air Sport Commissions. Note: in this section, the term "approved" means approved by the Air Sport Commission concerned.

7.1.3 Position. Position may be measured directly, by reference to approved maps, or by GNSS fix. If by GNSS fix, all fixes, points, locations, coordinates, and any maps concurrently used must be referenced to the WGS84 Earth Datum.

7.1.4 Distance. Distance may be measured directly or determined from approved maps.

7.1.5 Bearing. Bearing may be measured directly or determined from approved maps. The bearing at a point is the bearing from that point.

7.1.6 Time. Elapsed times and time of day may be measured either by approved timepieces or by GNSS.

7.1.7 Altitude. Pressure altitude may be measured using approved pressure-measuring devices. Geometric altitude and/or height above the surface may be measured using GNSS, optical methods, or radar.

7.1.8 Mass. Mass shall be determined using scales and methods approved by the Air Sport Commission concerned. The take-off mass of an aircraft shall be its total mass at take-off including flight crew.

7.2 Calculations

7.2.1 General. The methods and standards of precision for calculating Distance, Bearing, Altitude, Speed, and Scores shall be determined by each FAI Air Sport Commission and specified in the appropriate section of the Sporting Code. Note: in this section, the term "approved" means approved by the Air Sport Commission concerned.

7.2.2 Earth Model. The Air Sport Commissions are responsible for the specification of the basis of geometric calculations. If not otherwise specified by the Air Sport Commissions, the earth model to be used for geometric calculations shall be the WGS84 ellipsoid. If a sphere is specified, it shall be the "FAI Sphere." If a planar model is to be used, then the projection must be strictly defined.

7.2.3 Distance. If calculated from coordinates, distance shall be taken as the length of the geodesic on the earth model in use.

7.2.4 Bearing. If calculated from coordinates, bearing shall be taken as the initial bearing of a geodesic from a given point, on the earth model in use.

7.2.5 Altitude. The methods for calculations of corrections to measured altitudes (if required) shall be specified by the Air Sport Commissions. If a standard pressure model is required, it shall be the ICAO Standard Atmosphere.

7.2.6 Speed. Speed will be calculated from distances and elapsed times.

7.2.7 Scores. The methods for calculations of scores shall be specified by the Air Sport Commissions.

7.3 Margins

7.3.1 Each Air Sport Commission is responsible for specifying the margins by which a record claim must exceed an existing record, subject to paragraph 7.4.2 of this Chapter.

7.4 Approvals

7.4.1 As an alternative to specifying algorithms, each Air Sport Commission may meet its obligation to specify computational methods by approving specific flight evaluation and scoring programs. If this method is used, then the ASC must implement procedures for testing, approval, and version control of the flight evaluation and scoring programs.

7.4.2 The FAI Executive Board reserves the right to review the standards of certification and the methods of analysis of any international record claim.

GLOSSARY OF TERMS AND ABBREVIATIONS

This section amplifies a number of terms which are used in the main text and gives some generally accepted definitions and abbreviations relevant to air sports

Numerical

1	(Sporting Code Section) - Aerostats
2	(Sporting Code Section) - General Aviation
3	(Sporting Code Section) - Gliding
4	(Sporting Code Section) - Aeromodelling
5	(Sporting Code Section) - Parachuting
6	(Sporting Code Section) - Aerobatics
7	(Sporting Code Section) - Hang Gliding
8	(Sporting Code Section) - Astronautics
9	(Sporting Code Section) - Rotorcraft
10	(Sporting Code Section) – Microlights and Paramotors
11	(Sporting Code Section) - Human Powered Aircraft
12	(Sporting Code Section) - Unmanned Aerial Vehicles (AL5)

Other Documents Available

In addition to the sections of the Sporting Code (table, para 1.4), other Documents are available from FAI on request:

FAI Distance Calculations for Spherical Earth Model (Ex GS Annex B) (AL11)
FAI Anti-Doping Control Regulations for Air Sports (3.11.2.6 refers)
Rules for FAI WAG (3.1.7 refers)
International Jury Members Handbook (4.3.2.5 refers)
Technical Specification for IGC-approved GNSS Flight Recorders (*Although this document is maintained by IGC, other airsports may wish to use it, or parts of it such as the common data file standard which enables analysis programmes developed for it to be used. (AL 2)*

Alphabetical

A	(FAI Class) - Balloons
Aerodyne	See Chapter 2 for definitions, page 2 – 1
Aeronautics	For FAI purposes, aerial activity, including all air sports, equal to or less than 100 kilometres of the earth's surface (Source: Statutes, Preamble, Terms). See also under Space. (AL6)
Aerostat	See Chapter 2 for definitions, page 2 - 1
Aircraft	See Chapter 2 for definitions, page 2 - 1
AL	Amendment List (For the GS, takes effect on the 1 Jan following issue by FAI)
Altitude	The vertical distance from mean sea level (MSL). See also \`QNH', and \`Height'.
AMSL	Above Mean Sea Level
ASC	Air Sport Commission (List, page 1-2), responsible for a specific Sporting Code section.
AUW	All Up Weight / Mass
B	(FAI Class) - Airships/Dirigibles
C	(FAI Class) - Aeroplanes
C	(Temperature) – Celsius
CANS	Commission on Airspace and Navigation Systems. A technical commission of

	FAI. (AL13)
CAS	Calibrated Airspeed (IAS corrected for Instrument and Pressure Errors)
CASI	Commission d'Aéronautique Sportive Internationale (the Air Sport General Commission of FAI)
Certification	The signature on and preparation of certificates and other documents concerned with the process of flight verification with a view to validation of an FAI Flight Performance
CIA	Commission Internationale d'Aérostation, the International Ballooning Commission
CIACA	Commission Internationale des Amateurs Constructeurs d'Aéronefs, the FAI Amateur-built and Experimental Aircraft Commission. A technical commission of FAI. (AL7)
CIAM	Commission Internationale d'Aéromodélisme, the International Aeromodelling Commission
CIEA	Commission Internationale d'Education Aéronautique et Spatiale, the education commission. A technical commission of FAI. (AL1)
CIG	Commission Internationale de Giraviation, the International Rotorcraft Commission
CIMA	Commission Internationale de Micro-Aviation, the International Microlight and Paramotor Commission
CIMP	Commission Internationale Médico-Physiolgique, the medical commission. A technical commission of FAI. (AL1)
CIVA	Commission Internationale de Voltige Aerienne, the International Aerobatics Commission
CIVL	Commission Internationale de Vol Libre, the International Hang Gliding and Paragliding Commission (AL7)
C of A	Certificate of Airworthiness
Commission	FAI Commissions consist of Air Sport Commissions (ASC) and Technical Commissions. The ASC are listed on page 1-2 and each one is responsible for a specific section of the Sporting Code. Technical commissions consist of CIACA, CIEA, CIMP, EnvC, and CANS - see under these initials in this glossary. (AL13)
CP	Control Point
D	(FAI Class) - Gliders
DM	(FAI Class) - Motor Gliders
E	(FAI Class) - Rotorcraft (Helicopters and Autogyros)
Earth Model	**The mathematical surface upon which geometric calculations are performed. Earth models in use are ellipsoidal, spherical, and planar.**
Ellipsoid	**For FAI purposes, an ellipsoid is the surface formed by the rotation of an ellipse about its minor axis.**
EnvC	The Environmental Commission. A technical commission of FAI. (AL1)
F	(FAI Class) – Model Aircraft (AL7)
FAI	Fédération Aéronautique Internationale, with its headquarters in Lausanne, Switzerland.
FAI Sphere	**A sphere of radius 6371 kilometres, exactly.**
g	**Acceleration due to the force of gravity (9.81 m/sec^2)**
G	**The force on an object under acceleration expressed in multiples of g.**
G	(FAI Class) - Parachuting
GAC	General Aviation Commission
Geodesic	**The path of shortest length between two points on a surface.**
Geodetic Datum	**A specification of the shape, size and location in space of the surface of the Earth. Specification of the Geodetic Datum is necessary for unique GNSS solutions, and for map-making.** ***WGS84 (q.v.) is a geodetic datum.***
GLONASS	Global Orbital Navigation Satellite System, the Russian GNSS system similar to the US GPS
GNSS	Global Navigation Satellite System (Generic term for all systems such as the

	Russian GLONASS and the US GPS)
GNSS fix	**The 4-dimensional (latitude, longitude, altitude, UTC) location of a point in space and time, as determined by a GNSS**
GPS	Global Positioning System (US GNSS System managed by the Departments of Defense and Transportation)
H	(FAI Class) - Vertical Take-off and Landing Aircraft
Height	The vertical distance from a given height datum such as the take-off place. See also `QFE', and `Altitude'.
Homologation	The validation of a Flight Performance by an NAC or FAI for record purposes
Host NAC	The NAC of a country in which an FAI Sporting Event is organized (AL11)
HPa	Hecto Pascal (Pressure unit, equal to a millibar)
I	(FAI Class) - Human Powered Aircraft
IAS	Indicated Airspeed
ICAO	International Civil Aviation Organisation (HQ in Montreal, Canada)
ICARE	International Commission for Astronautics Records
IGC	International Gliding Commission
IPC International Parachuting Commission	
ISA	**International Standard Atmosphere, as defined by ICAO. Reference: *Manual of the ICAO Standard Atmosphere (extended to 80 kilometres (262 500 feet))*, Doc 7488-CD, Third Edition, 1993, ISBN 92-9194-004-6**.
K	(FAI Class) - Spacecraft
M	(FAI Class) - Tilt-Wing Aircraft
MG	Motor Glider (FAI Class)
min	Minute, unit of time (UT), compared to `arcmin' which is 1 minute of angle
m/s	Metres per Second
MSL	Mean Sea Level
N	(FAI Class) - STOL Aircraft
NAC	National Airsport Control
O	(FAI Class) - Hang Gliders and Paragliders
O&R	Out and Return
Obligations	(such as to FAI) Obligations of NACs to FAI are listed in the FAI Statutes, search for the word "obligations". At the time of finalising this edition of the GS the relevant Statute number was 2.4.2.2. (AL6)
OO	Official Observer
Organizer	The event organizer approved by, and acting with or on behalf of, an NAC or the FAI (AL11)
Ornithopter	A machine that achieves and sustains flight by the sole means of flapping wings.
P	(FAI Class) - Aerospacecraft
QFE	**Altimeter pressure setting that results in an indication of zero on the surface**
QNH	**Altimeter pressure setting that results in an indication of height above sea level**
R	(FAI Class) - Microlights, Powered Hang Gliders and Paramotors
S	(FAI Class) - Space Models
Shall	See under `Wording'
Should	See under `Wording'
Soaring	The utilisation of the vertical component of movements of air in the atmosphere for the purpose of sustaining flight, without the use of thrust from a means of propulsion.(AL6)
Space	For FAI purposes, activities above 100 kilometres from the earth's surface. See also under Aeronautics. (AL6)
Sphere	**(delete)**

STOL Short TakeOff and Landing
TAS True Air Speed
Technical Commission See under Commission. (AL1)
TP Turn Point
U (FAI Class) – Unmanned Aerial Vehicle (AL5)
UT UTC to the local hour convention
UTC Universal Time Co-ordinated (ex-GMT)
Validation An act of ratification or official approval. In FAI terms, the act of approving a Flight Performance (or an element of one such as reaching a Turn Point) for FAI purposes.
Verification The process of checking and assembling evidence with a view to validating a Flight Performance
Vincenty method An empirical method used to calculate the distance between pairs of points on the WGS84 ellipsoid.
Reference: http://www.ngs.noaa.gov/PUBS_LIB/inverse.pdf
Example: http://www.fai.org/how-to-set-a-record/world-distance-calculator
Vs Stalling Speed
VTOL Vertical TakeOff and Landing
WADA - World Anti Doping Agency. See http://www.wada-ama.org (AL8)
WAG - World Air Games. An international sporting event involving several FAI air sports at the same time, see GS3.1.7. (AL8)
WGS84 Earth Datum See WGS84
WGS84 World Geodetic System 1984 – For FAI purposes, this is the standard Geodetic Datum.
WGS84 Ellipsoid An *ellipsoid* (q.v) based on an ellipse with a semi-minor axis of 6356,7523 kilometres and a semi-major axis of 6378,1370 kilometres. The minor axis is the polar axis.
Wording The use of "shall" and "must" implies that the aspect concerned is mandatory; the use of "should" implies a non-mandatory recommendation; "may" indicates what is permitted and "will" indicates what is going to happen. Words of masculine gender should be taken as including the feminine gender unless the context indicates otherwise. *Italics are used for explanatory notes.*

General Recommendations

Note: this section addresses topics that are not strictly within the Terms of Reference of this Working Group. However, the following non-mathematical opinions are shared by the members of the WG, all of whom have been "students of the Sporting Code" for a very long time.

1. Technical
Beyond the recommendations made in this report, we believe that the General Section would benefit from a general technical review, especially in the Glossary. Science and engineering concepts should be more rigorous. CASI should review the technological currency of the General Section. Do we need to specify the handling of photographic film? Should Observation Zones be the domain of the General Section, or of the ASCs?

2. Procedural
We recommend that CASI review the process that results in technical changes to the General Section. The 2012 changes came as a surprise to the mathematical advisors, programmers, and rule-writers in several Commissions. If CASI had sought expert advice beforehand, or had notified the Commissions of the intent to make a far-reaching technical change, then the situation in which we find ourselves today might have been avoided.

3. Editorial
The General Section shows a serious lack of proofreading. It still describes a Technical Commission that no longer exists; it uses a hybrid of British and American English; and its Table of Contents is no longer accurate (GS 7.3.1.1 is a good example!) The General Section should be edited to be more self-consistent.

4. Cooperative
The members of this WG benefited greatly from the education we received from each other in the matter of solutions to common technical problems. We believe that CASI should consider forming a more permanent technical WG or Subcommittee charged with finding convergent solutions when appropriate.

Conclusion

In 1994 FAI was among the first aeronautical organisations to recognise the need for rationalisation of navigational solutions, waypoints, flight logging, and airspace definitions. FAI led the world in the now-universal adoption of the WGS84 Geodetic Datum as the basis for geographic reference.

Since then, however, FAI has not done an admirable job of staying current with the challenges of combining the new technology with appropriate mathematical methods in the regulation of Air Sports. Chapter 7 of the General Section is founded on what appears to be a weak understanding of analytic geometry.

The 2012 amendment to Chapter 7 (AL16) has put seven Air Sports out of compliance with the General Section and put them in the position of not being able to comply. It has also opened the door to the criticism contained in this report:

- Chapter 7 has strayed from its original purpose. Much of the contents of the Chapter are off-topic.

- Chapter 7 has blurred the distinction between a measurement and a calculation, giving the impression that FAI does not understand the difference.

- Chapter 7 contradicts itself in the assignment of areas of responsibility. It asserts that a single earth model can be used "for FAI purposes," without consideration of the differences between records, awards, and competitions, and without regard for the special computational needs of each Air Sport.

- The entries in the General Section Glossary related to Chapter 7 are incomplete and mathematically dubious.

The original purpose of Chapter 7 can be preserved by replacing it with the reorganised and clarified version contained in this report.

Submitted August 3, 2013 by the CASI Mathematics Working Group

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Appendix A

Earth models in current use by flight evaluation programs

CIA
The sphere is specified for all calculations for records, and for live tracking of the Coupe Gordon Bennett Race. Approved electronic and/or printed maps are used in competitions.

CIMA
The ellipsoid is used for records. Competition flights (nav tasks) are evaluated with a program (MicroFLAP) that uses both the ellipsoid and a flat plane earth model. Other programs that are used (Coutraci, SeeYou, OziExplorer) use a combination of all three earth models (ellipsoid, sphere, flat plane).

IGC
The ellipsoid is used for badges and records. For competitions, use of the sphere is specified in Sporting Code Section 3, Annex A.

CIVL
The ellipsoid is specified for records. Use of the sphere is specified in "CIVL GAP 2012," the FAI document that governs scoring for Hang-Gliding and Paragliding.

CIG
The ellipsoid is used for records. Competition flights are evaluated with an approved program (AFLOS Heli) that uses a flat plane Earth Model.

GAC
The ellipsoid is used for records. Competition flights are evaluated by a suite of programs (AFLOS, FSNavigator, OziExplorer, FlyNavigation, etc.) that use all three Earth Models: ellipsoid, sphere, flat plane.

ICARE
The von Karman ellipsoid is used for space records.

Appendix B

Calculation of Cross-Track Error

This is a summary of the "XTE Problem" as it relates earth models.

B1. Problem Definition

It takes a few words to define the problem. The calculation of Cross-Track Error (XTE) is a classic navigation problem. It is an example of the general problem of calculating the distance and bearing from a point to a line. In this context, a "line" is taken to be a geodesic (the path of shortest length between two points on a surface).

The statement of the general problem is:

> Given a particular earth model, how do we calculate the distance and bearing from a point to a geodesic?

In this appendix, we will refer to this as the "XTE Problem."

B2. Earth Models

The "earth model" is the surface upon which geometric calculations are made. There are three earth models in common use. In all three, points are specified by latitude and longitude in degrees. Calculated distances are usually given in kilometres, and bearings in degrees True.

B2.1 <u>Planar</u>

In planar models, the surface of the Earth is approximated by a flat plane. The coordinate system is a modified Cartesian coordinate system. The x-axis is a parallel and the y-axis is a meridian. To minimise error, the origin is placed in the vicinity of the calculation, for each calculation. Because the meridians converge at the poles, the x-axis is scaled according to the local latitude. To arrive at the desired distance units, a scale factor is required. A common example of a planar earth model is UTM.

B2.2 <u>Spherical</u>

The Spherical earth model is simply a sphere with a specified radius.

B2.3 <u>Ellipsoidal</u>

The Ellipsoidal earth model is a surface formed by rotating an ellipse about its minor axis, which is taken to be the polar axis. The lengths of the semi-major and semi-minor axes are specified.

All three earth models are approximations of the true shape of the Earth. When considering the entire planet, the ellipsoid is the most accurate of the three.

B3. Available Methods

B3.1 In the Planar earth model, all the methods of plane analytic geometry and trigonometry are available. Geodesics are straight lines.

B3.2 In the Spherical earth model, all the methods of spherical analytic geometry and trigonometry are available. In this case, however, geodesics are Great Circles.

B3.3 In the Ellipsoidal earth model, no analytic methods are available. One empirical method (and its inverse) is available. The method is the "Vincenty method," and it is used to calculate the distance and bearing from one specified point to another specified point. The Vincenty method is accurate for all practical purposes, but its complexity must be considered in the performance of real-time computations.

B4. Solving the XTE Problem on the Plane

The Pythagorean Theorem and plane trigonometry provide us with analytic expressions for distance and bearing between points. Triangulation can be used to derive the formula for the answer.

B5. Solving the XTE Problem on the Sphere

Similarly, formulas are available for distance and bearing on the sphere, and a formula for distance and bearing to a great circle can be derived.

B6. Solving the XTE Problem on the Ellipsoid

As noted above, there are no analytical methods associated with the ellipsoid. To our knowledge, there are no empirical methods for finding the coordinates of a point on a geodesic with a specified bearing to another point. This leaves two possible approaches to solving the XTE Problem:

a) Change to one of the other two earth models (plane or sphere), solve the problem there, and assume that the error is negligible.

b) Use an iterative numerical method (based on Vincenty or a similar empirical method) and a criterion for deciding when the residual error is sufficiently small.

Both approaches are problematical. Switching earth models in the middle of a computation leads to paradoxes caused by the simultaneous existence of two different ways to do a calculation. Iterative numerical methods are usually difficult to define, and are often infeasible for real-time calculations, even with powerful computers.